DECOMPOSITIONS OF TOPOLOGICAL SETS

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Introduction

α-δ-open sets, pre-δ-open sets, semi-δ-open sets, strongly β-δ-open sets and b-δ- open sets in topological spaces were studied by [ ] respectively. In this paper, we introduce new classes of sets by using losed sets in topological spaces and study their basic properties; and their connections with other types of topological sets. Moreover some new decompositions of topological sets are obtained.

Preliminaries

We recall the following definitions which are useful in the sequel.

Definition 2.1. A subset A of (X, τ) is called

(1) α-open [10] if A ⊆int(cl(int(A)));
(2) preopen [9] if A ⊆int(cl(A));
(3) semi-open [7] if A ⊆ cl(int(A));
(4) β-open [1] if A ⊆cl(int(cl(A)));
(5) b-open [2] if A ⊆int(cl(A))∪cl(int(A)).

The family of all α-open (resp. preopen, semi-open, β-open, b-open) sets of X is denoted by αO(X) (resp. PO(X), SO(X), βO(X), BO(X)).

Definition 2.2. A subset A of (X, τ) is called

(1) a Λ-set if A = $A^{\wedge}$ where $A^{\wedge}$= ∩ {G : A ⊆ G, G ∈τ} [8].
(2) a $\wedge_{\alpha}$-set if A = Λα(A) where $\wedge_{\alpha}$ (A) = ∩{G : A ⊆ G, G ∈ αO(X)} [5].
(3) a $\wedge_{s}$-set if A = Λs(A) where $\wedge_{s}$ (A) = ∩{G : A ⊆ G, G ∈ SO(X)} [4].
(4) a $\wedge_{p}$-set if A = Λp(A) where $\wedge_{p}$ (A) = ∩{G : A ⊆ G, G ∈ PO(X)} [4].
(5) a $\wedge_{\beta}$-set if A = Λβ(A) where $\wedge_{\beta}$ (A) = ∩{G : A ⊆ G, G ∈βO(X)} [11].
(6) a $\wedge_{b}$-set if A = Λb(A) where $\wedge_{b}$ (A) = ∩{G : A ⊆ G, G ∈ BO(X)} [6].

Definition 2.3. A subset A of (X, τ) is called λ-closed [3] if A =L ∩ F, where L is a Λ-set and F is closed.

Characterizations of generalized $\lambda-\delta-\alpha$ closed sets

Definition 3.1:

A subset A of $(X\ ,\tau)$ is called $a\ C_{\delta}$ set if $A = L \cap F$, where L is open and $F$is $pre-\delta-$ closed set.

A subset A of (X, τ) is called $a\ BC_\delta$ set if $A = L \cap F$, where L is open and $F$is $b - \delta -$ closed set.

A subset A of (X, τ) is called $an\ \eta_\delta$ set if $A = L \cap F$, where L is open and $F$is $\alpha - \delta -$ closed set.

Definition 3.2

A subset A of $(X, \tau)$is called $-\delta - \alpha$ closed if$A = L \cap F$, where L is a set and $F$is $\alpha - \delta$closed set.

A subset A of (X, τ) is called a DC_δ- set if if$A = L \cap F$, where L is open and F is $semi\ \delta$ closed set.

A subset A of (X, τ) is called a sFC_δ- set if if$A = L \cap F$, where L is open and $F$is $strongly\ \beta - \delta$ closed set.

Lemma 3.3

In (X, τ) , the following statements hold.

Every $\alpha - \delta$ closed set is $\lambda - \delta - \alpha$ closed.

Every λ set is $\lambda - \delta - \alpha$ closed.

Example 3.4

Let $X = \{a, b, c, d\}$ and $= \{\Phi, X, \{d\}, \{a, c\}, \{a, c, d\}\}$. Then

$\{d\}$is $\lambda - \delta. -\alpha$ closed but not $\alpha - \delta$ closed set.

{a,b,c} is $\lambda - \delta - \alpha$ closed set but not λ set.

Lemma 3.5

For a subset A of (X, τ), the following conditions are equivalent

A is $\lambda - \delta - \alpha$ closed

$A = L \cap \alpha cl_\delta(A)$, where L is a λ set.

$A = A^\lambda \cap \alpha cl_\delta(A)$.

Lemma 3.6

In (X, τ) , the following statements hold.

Every $\alpha - \delta$ closed set is η_δ set.

Every η_δ set is $\lambda - \delta - \alpha$ closed.

Example 3.7. In Example 3.4, $\{d\}$ is an η_δ-set but not $\alpha - \delta$ closed.

Definition 3.8. A subset A of (X, τ) is said to be

$\delta_{\alpha g}$-closed if $\alpha cl_\delta(A)\ \ U$ whenever $A\ \ U$ *and* U *is open.*

δ_{gs}-closed if $scl_\delta(A)\ \ U$ whenever $A\ \ U$ *and* U *is open.*

δ_{gp}-closed if $pcl_\delta(A)\ \ U$ whenever $A\ \ U$ *and* U *is open.*

δ_{gsp}-closed if $s\beta cl_\delta(A)\ \ U$ whenever $A\ \ U$ *and* U *is open.*

δ_{gp}-closed if $bcl_\delta(A)$ U whenever A U *and* U *is open*.

Lemma 3.9. A subset A of $(X, \tau)$is $\delta_{\alpha g}$-closed if and only if $\alpha cl_\delta(A)$ A^λ .

Theorem 3.10. For a subset A of (X, τ), the following conditions are equivalent.

A is $\alpha - \delta$closed.

A is $\delta_{\alpha g}$-closed and an η_δ-set.

A is $\delta_{\alpha g}$-closed and $\lambda - \delta - \alpha$ closed.

Proof. (1) ⇒(2) and (2) ⇒ (3): Obvious.

(3) ⇒ (1): Since A is $\delta_{\alpha g}$-closed, by Lemma 3.18, $\alpha cl_\delta(A)$ A^λ . Since A is $\lambda - \delta - \alpha$closed, by Lemma 3.9, $A = A^\lambda \cap \alpha cl_\delta(A) = \alpha cl_\delta(A)$. Hence A is $\alpha - \delta$closed.

Remark 3.11. The following example shows that the concepts of $\delta_{\alpha g}$-closed sets and η_δ −sets are independent of each other.

Example 3.12.(1) In Example 3.4, $\{d\}$ is an η_δ-set but not $\delta_{\alpha g}$-closed.

(2) In Example 3.4, $\{b, c, d\}$ is $\delta_{\alpha g}$-closed but not an η_δ-set.

Remark 3.13. The following example shows that the concepts of $\delta_{\alpha g}$-closed sets and $\lambda - \delta - \alpha$ closed sets are independent of each other.

Example 3.14. (1) In Example 3.4, $\{d\}$ is $- \delta - \alpha$ closed but not $\delta_{\alpha g}$closed.

(2) In Example 3.4, $\{a, b, c\}$ is $\delta_{\alpha g}$-closed but not $\lambda - \delta - \alpha$ closed.

Definition 3.15. A subset A of a (X, τ) is called

$\lambda - \delta$ −semi-closed if $A = L \cap F$, where L is a λ set and $F$is $semi - \delta$ −closed set.

$-\delta$ −pre-closed if $A = L \cap F$, where L is a set and $F$is $pre - \delta$ −closed set.

$-\delta - \beta$ −closed $A = L \cap F$, where L is a set and $F$is $strongly\ \beta - \delta$ −closed set.

$-\delta - b$ - closed if $A = L \cap F$, where L is a set and $F$is $b - \delta$ −closed set.

Lemma 3.16. A subset A (X, τ) is

δ_{gs}-closed if and only if $scl_\delta(A)$ A^λ .

δ_{gp}-closed if and only if $pcl_\delta(A)$ A^λ.

δ_{gsp}-closed if and only if $s\beta cl_\delta(A)$ A^λ.

δ_{gb} −closed if and only if $bcl_\delta(A)$ A^λ .

Corollary 3.17. For a subset A of a (X, τ), the following conditions are equivalent

(i) A is semi−δ −closed.

A is δ_{gs}-closed and a DC_δ -set.

A is δ_{gs}-closed and $\lambda-\delta$ −semi-closed.

(i) A is pre$-\delta$ −closed

A is δ_{gp} −closed and a C_δ −set.

A is δ_{gp} −closed and $\lambda-\delta$ −pre-closed.

(i) A is strongly $\beta-\delta$ −closed

A is δ_{gsp} −closed and a sFC_δ −set

A is δ_{gsp} −closed and $\lambda-\delta-\beta$ −closed.

(i) A is $b-\delta$ −closed

A is δ_{gb} −closed and a BC_δ −set.

A is δ_{gb} −closed and $\lambda-\delta-b$ −closed.

Proof. The proof is similar to that of Lemma 3.5 and Theorem 3.10.

Remark 3.18. The following examples show that the concepts of

δ_{gs} −closed sets and DC_δ −sets are independent of each other.

δ_{gs} −closed sets and $\lambda-\delta$ −semi-closed sets are independent of each other.

δ_{gp} −closed sets and C_δ −sets are independent of each other.

δ_{gp} −closed sets and$\lambda-\delta$ −pre-closed sets are independent of each other.

δ_{gsp} −closed sets and sFC_δ −sets are independent of each other.

δ_{gsp} −closed sets and $\lambda-\delta-\beta$ −closed sets are independent of each other.

δ_{gp} −closed sets and BC_δ −sets are independent of each other.

δ_{gp} −closed sets and $\lambda-\delta-b$ −closed sets are independent of each other.

Example 3.19.

(1) In Example 3.4, $\{a,c,d\}$ is a DC_δ −set but not δ_{gs} −closed.

(2) In Example 3.4, $\{c\}$is δ_{gs} −closed but not a DC_δ −set.

Example 3.20.

(1) In Example 3.4, $\{a,c,d\}$ is $\lambda-\delta$ −semi-closed but not δ_{gs} −closed.

(2) In Example 3.4, $\{a,b\}$is δ_{gs} −closed but not $\lambda-\delta$ −semi-closed.

Example 3.21.

Let $X=\{a,b,c,d\}$and $=\{\Phi,X,\{a\},\{b\},\{a,b\}\{b,c\},\{a,b,c\}\}$. Then

Example 3.22.

(1) In Example 3.21, $\{a\}$ is $\lambda-\delta$ −pre-closed but not δ_{gp} −closed.

(2) In Example 3.21, $\{a,d\}$is δ_{gp}closed but not $\lambda-\delta$ −pre-closed.

Example 3.23.

(1) In Example 3.21, $\{a, b, c\}$ is a sFC_{δ} −set but not δ_{gsp} −closed.

(2) In Example 3.21, $\{d\}$is δ_{gsp} −closed but not a sFC_{δ} −set.

(2) In Example 3.21, $\{d\}$is δ_{gp} −closed but not a BC_{δ} −set.

Example 3.26.

(1) In Example 3.21, $\{a, b\}$ is $\lambda - \delta - b$ −closed but not δ_{gp} −closed.

(2) In Example 3.21, $\{d\}$is δ_{gp} −closed but not $\lambda - \delta - b$ −closed.

Definition 3.27.

A subset A of a (X, τ) is called

(1) $\lambda - \delta - \alpha g^{*}$ −closed if $A = L \cap F$, where L is a λ_{α} set and F is δ −closed .

(2) $\lambda - \delta - sg^{*}$ −closed if$A = L \cap F$, where L is a λ_{s} set and $F$is δ −closed .

(3) $\lambda - \delta - pg^{*}$ −closed if $A = L \cap F$, where L is a λ_{p} set and $F$is δ −closed .

(4) $\lambda - \delta\beta g^{*}$ −closed if $A = L \cap F$, where L is a λ_{β} set and $F$is δ −closed .

(5) $\lambda - \delta - bg^{*}$ −closed if$A = L \cap F$, where L is a λ_{b} set and $F$is δ −closed .

Definition 3.28.A subset A of (X, τ) is called

(1) an $\delta - \alpha lc$ −set if $A = L \cap F$, where L is a α open and F is δ −closed .

(2) an $\delta - slc$ −set if $A = L \cap F$, where L is a semi- open and F is δ −closed .

Lemma 3.29.

Every λ_{α} −set (resp. λ_{s} −set, λ_{p}set, λ_{β}-set, λ_{b}set) is $\lambda - \delta - \alpha g^{*}$ −closed (resp. $\lambda - \delta - sg^{*}$ −closed, $\lambda - \delta - pg^{*}$ −closed, $\lambda - \delta - \beta g^{*}$ −closed, $\lambda - \delta - bg^{*}$ −closed).

Example 3.30.

Let $X = \{a, b, c\}$and= $\{\Phi, X, \{a\}, \{b\}, \{a, b\}\}$. Then {c} is $\lambda - \delta - \alpha g^{*}$ −closed but not a λ_{α} −set.

Example 3.31.

Let $R, N, Q^{*}$and Q the set of real numbers, the set of natural numbers, the set of irrational numbers and the set of rational numbers respectively. Suppose= $\{\Phi, R, N, Q^{*}, Q^{*} \cup \mathrm{N}\}$. Then $H = Q \cap R \setminus N$ is δ-closed and hence $\lambda - \delta - sg^{*}$ −closed but not λ_{s} −set.

Example 3.32.

Let $X = \{a, b, c\}$and$\tau = \{\Phi, X, \{a\}, \{c\}, \{a, c\}\}$. Then {b} is $\lambda - \delta - pg^{*}$ −closed but not a λ_{p} −set.

Example 3.33.

In Example 3.31, $H = Q \cap R \backslash N$ is δ-closed and hence $\lambda - \delta - \beta g^*$ −closed but not λ_β −set.

Example 3.34.

In Example 3.31, H=Q\RnN is -closed and hence $\lambda - \delta - bg^*$ −closed but not λ_b −set.

Lemma 3.44.

For a subset A of a (X , τ), the following conditions are equivalent.

(1) A is $\lambda - \delta - \alpha g^*$ −closed.

(2) $A = L \cap cl_\delta(A)$, A where L is a λ_α −set.

$A = \lambda_\alpha(A) \cap cl_\delta(A)$

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